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Spontaneous Lorentz violation and nonpolynomial interactions

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Abstract

Gauge-noninvariant vector field theories with superficially nonrenormalizable nonpolynomial interactions are studied. We show that nontrivial relevant and stable theories have spontaneous Lorentz violation, and we present a large class of asymptotically free theories. The Nambu–Goldstone modes of these theories can be identified with the photon, with potential experimental implications.

1. Introduction

Experiments show that nature is well described at presently accessible energies by two field theories: the Standard Model (SM) of particle physics, and Einstein's General Relativity. These are expected to arise as the low-energy limit of a fundamental theory of quantum gravity at the Planck scale, $M_P \simeq 10^{19}$ GeV. The discrepancy between M_P and attainable energies makes experimental signals from this underlying theory difficult to identify, but one promising class of observables involves violations of Lorentz symmetry arising from new physics at the Planck scale.[1]

An interesting and challenging issue that has received little attention to date is the extent to which Lorentz violation might be generic or even ubiquitous in prospective fundamental theories. The present work initiates a study of this issue. For definiteness, we focus attention on the elegant possibility that Lorentz symmetry is spontaneously broken in the underlying theory [6]. The basic idea is that interactions in the underlying theory induce nonzero vacuum expectation values for one or more Lorentz tensors, which can be regarded as background quantities in the vacuum throughout spacetime.

The analysis in this work adopts the methods of Lagrangian-based quantum field theory, in which observable effects of Lorentz violation are described by an effective low-energy field theory [7–9], and it assumes that the issue of obtaining the required hierarchy [7,10,4] for the associated coefficients for Lorentz

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[1] For a variety of recent reviews see, e.g., Refs. [1–5].

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violation can be addressed. In this language, the fundamental theory may have many types of fields and interactions, including ones that are nonrenormalizable at the level of the effective low-energy theory. A comprehensive study of the likelihood of Lorentz violation at this level appears infeasible at present. However, in Lorentz-invariant scalar field theories, certain nonpolynomial and hence superficially nonrenormalizable interactions have been shown to be relevant in the sense of the renormalization group (RG) by studying the natural cutoff dependences of the coupling constants [11]. The Gaussian fixed point of the RG flow is ultraviolet-stable along certain directions in the parameter space of interactions, and these directions correspond to nontrivial asymptotically free theories. Here, we exploit this idea by generalizing the scalar analysis to the case of vector fields and investigating the occurrence of spontaneous Lorentz violation in the resulting theories.

The prototypical field theories for Lorentz violation with a vector field B^μ are the so-called bumblebee models [12].[2] These involve a gauge-noninvariant potential $V(B^\mu B_\mu)$ that has a minimum at nonzero B^μ inducing spontaneous Lorentz violation. We consider a generic model of this type with a conventional Maxwell-type kinetic term and an arbitrary nonpolynomial potential, as might arise from a fundamental theory, and we take RG relevance of the interactions and stability of the associated quantum field theory as practical criteria determining acceptable models for our study. These assumptions make it possible to address the ubiquity of Lorentz violation in a definite context. Surprisingly, we find that consistent stable relevant theories of this type *must* have spontaneous Lorentz violation and that a large class of such theories arises from superficially nonrenormalizable bumblebee models. Moreover, these theories naturally contain Nambu–Goldstone (NG) modes [25] associated with spontaneous Lorentz violation that can be identified with the photon, a result with potential experimental consequences.

To perform the analysis, we study the flow in the Wilson formulation of the RG [26], in which the theory is considered with a momentum cutoff. An analysis with a more general cutoff is also possible [27], through the use of the Polchinski formulation of the RG [28]. Since any Lorentz violation in nature is a weak effect, we restrict attention to the linearized form of the RG transformation, in which only terms that are first-order in the interaction are retained. The literature for the Lorentz-invariant scalar case contains some discussion about the persistence of the nonpolynomial interactions when the full nonlinear RG is considered [29]. However, a nonperturbative demonstration to the contrary would require showing that the nonpolynomial potentials can be expanded as a sum of an infinite number of irrelevant RG modes, a challenging task. Moreover, there is evidence for persistence: in the limit where the number of scalar-field components is large, it is known that the RG equations for the nonpolynomial modes can be integrated into a region far from the fixed point [30]. This suggests that the novel potentials exist outside the linearized regime for finite-component fields as well. Other generalizations of the original results include Refs. [31,32] and a study of the impact of Lorentz violation on asymptotically free scalar and spinor field theories [33]. No evidence exists for relevant nonpolynomial theories involving spinor fields, but a leading-order analysis shows that Lorentz violation is a prerequisite for their existence, a conclusion compatible with the results for vector fields obtained below.

2. Running the bumblebee

Consider a theory for a vector-valued 'bumblebee' potential field B^μ with Lagrange density

$$\mathcal{L} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} + V\big(B^\mu B_\mu\big), \tag{1}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the field strength. The potential V is assumed to be representable as a power series in B^2, and it violates gauge invariance. For the simple case $V = m^2 B^2/2$, the theory describes a free massive vector boson. Potentials such as $V = \lambda(B^2 - b^2)^2/4$ for constant b^2 produce bumblebee models describing spontaneous Lorentz violation. Here, we consider a theory with more general nonpolynomial V. In what follows, we introduce a momentum cutoff Λ representing the only scale in the system, and we insert appropriate powers of Λ in V to render dimensionless all the couplings [11].

[2] Recent literature includes Refs. [9,13–24].

To study the RG flow for the theory (1), it is convenient to Wick rotate the variables and operate in Euclidean space. For scalar fields, the Euclidean RG equations are known [34]. The RG calculations for the Wick-rotated vector field parallel those for an SO(4) multiplet of four scalar fields, except for a minor modification arising from the structure of the kinetic term. At tree level, the transversality of the bumblebee kinetic term ensures there are only three propagating modes contained in the four-component field B^μ. This feature remains true in the RG analysis because no kinetic contributions to the two-point function for the fourth mode can arise. The relevant diagrams either have both external legs on the same vertex, yielding a tadpole and no kinetic contribution, or they have an internal line for the fourth mode, which vanishes. Since only three modes propagate rather than four, we must replace the zero-separation scalar propagator

$$\Delta_F^{jk}(0) = \int\limits_{|p|<\Lambda} \frac{d^4p}{(2\pi)^4}\frac{\delta^{jk}}{p^2} \tag{2}$$

with the transverse propagator

$$\begin{aligned} D_F^{\mu\nu}(0) &= -\int\limits_{|p|<\Lambda} \frac{d^4p}{(2\pi)^4}\frac{(\delta^{\mu\nu}-p^\mu p^\nu/p^2)}{p^2} \\ &= -\frac{3}{4}\Delta_F^{\mu\nu}(0) = -\frac{3\Lambda^2}{64\pi^2}\delta^{\mu\nu}. \end{aligned} \tag{3}$$

However, the extra factor of 3/4 relative to the scalar case contributes only to the overall normalization of the vector field.

The asymptotically free solutions of the linearized RG equations are [11]

$$V_\kappa\big(B^2\big) = g\Lambda^4\big[M(\kappa-2;2;z)-1\big]. \tag{4}$$

Here, $M(\alpha;\beta;z)$ is the confluent hypergeometric (Kummer) function [35],

$$M(\alpha;\beta;z) = 1+\frac{\alpha}{\beta}\frac{z}{1!}+\frac{\alpha(\alpha+1)}{\beta(\beta+1)}\frac{z^2}{2!}+\cdots, \tag{5}$$

with $z=-32\pi^2B^2/3\Lambda^2$. The parameter κ in Eq. (4) describes the growth of the coupling constant g when the cutoff scale Λ is changed. If all modes with momenta in the range $\Lambda_1<|p|<\Lambda_0$ are integrated out of the theory and the fields rescaled accordingly, then the renormalized g shifts to $g(\Lambda_0/\Lambda_1)^{2\kappa}$. Asymptotically free theories must have $\kappa>0$, and only these theories have nontrivial continuum limits. It is convenient to parametrize the coupling g as

$$g = \frac{c}{\kappa-2}, \tag{6}$$

so the sign of c gives the sign of the slope of V_κ at $z=0$.

Substantial additional complexities arise when the theory in Euclidean space is reconverted to Minkowski spacetime. For the scalar-field case, $\phi^2=\sum_{j=1}^4(\phi^j)^2$ is guaranteed to be positive, but the analogous quantity $-B^2=-B^\mu B_\mu$ for vector fields can be either positive (spacelike B^μ) or negative (timelike B^μ). This complicates the analysis of the stability of these theories. Furthermore, any nontrivial, stable, asymptotically free theory of this type necessarily involves spontaneous Lorentz breaking. This follows because z can now be positive or negative. If V_κ either increases or decreases at $z=0$, then there is a state with nonzero B^μ having lower energy than a state with $B^\mu=0$, so B^μ develops a Lorentz-violating vacuum expectation value. If instead V_κ has a vanishing derivative at $z=0$, then $c=0$, the potential vanishes identically, and the theory is trivial.

3. Stability analysis

To determine which κ correspond to stable theories, we examine the asymptotic behavior of V_κ as $z\to+\infty$ and $z\to-\infty$. For large positive z, the asymptotic formula

$$M(\alpha;\beta;z)\approx\frac{\Gamma(\beta)z^{\alpha-\beta}e^z}{\Gamma(\alpha)} \tag{7}$$

holds, with all corrections being suppressed by powers of z^{-1}. Also useful is the Kummer formula

$$M(\alpha;\beta;-z)=e^{-z}M(\beta-\alpha;\beta;z), \tag{8}$$

which is an exact relation.

Consider first the case of a spacelike expectation value for B^μ, so that the minimum of V_κ has $-B^2>0$ and hence $z>0$. This parallels the case of positive vacuum expectation value for ϕ^2 [11]. Suppose $c<0$. The potential is then decreasing with z at $z=0$, so if V_κ diverges to positive infinity for large z then at least one stable minimum must exist for $z>0$. Eq. (7) shows V_κ indeed diverges as $z\to+\infty$, and its sign is determined by the sign of $c/(\kappa-2)\Gamma(\kappa-2)$ or, equivalently, by the sign of $-\Gamma(\kappa-1)$. This is positive when κ lies in one of the open intervals $(0,1)$, $(-2,-1)$, $(-4,-3)$, etc. Since a nontrivial theory must have $\kappa>0$, the only relevant range is $0<\kappa<1$. In contrast, if $c>0$, then there are no stable potentials with local minima on the positive z-axis because either $V_\kappa\to-\infty$ as $z\to+\infty$ or V_κ increases monotonically.

The potentials V_κ of interest that generate a spacelike expectation value for B^μ are therefore those with $c<0$ and $0<\kappa<1$, corresponding to $g>0$ in the range $|c/2|<g<|c|$. The stability of V_κ for $z<0$ must, however, also be verified. For negative z,

$$V_\kappa(B^2)=g\big[e^{-|z|}M\big(4-\kappa;2;|z|\big)-1\big]. \tag{9}$$

In the range $0<\kappa<1$, the hypergeometric function grows faster with $|z|$ than $e^{|z|}=M(2;2;z)$. Since $g>0$, it follows that V_κ is positive for all negative z. These theories are therefore stable, with a spacelike expectation value for B^μ and spontaneous Lorentz violation.

Next, consider the case of a timelike expectation value for B^μ, for which the vacuum has $-B^2<0$ and $z<0$. Any stable theories of this type must have $\kappa\geqslant 1$, and also $c>0$ is required for stability as $z\to+\infty$. Moreover, as $z\to-\infty$ the asymptotic behavior of the hypergeometric function is determined by

$$\frac{c}{\kappa-2}e^{-|z|}M\big(4-\kappa;2;|z|\big)\approx\frac{c}{\kappa-2}\frac{\Gamma(2)|z|^{2-\kappa}}{\Gamma(4-\kappa)}. \tag{10}$$

For $1\leqslant\kappa<2$, this diverges to negative infinity for large negative z, so the potentials in this range are unstable. For $\kappa=2$, the potential vanishes. For $\kappa>2$, we find $V_\kappa\to-c/(\kappa-2)$ as $z\to-\infty$, since $|z|^{2-\kappa}\to 0$. The theory is therefore stable if there exists a $z<0$ for which $V_\kappa\leqslant-c/(\kappa-2)$, which occurs if and only if $M(4-\kappa;2;|z|)$ has a root. This function cannot have a root unless $\kappa>4$ because otherwise all the terms in the sum (5) are positive. However, $M(\alpha;2;|z|)$ does indeed have a root for α sufficiently large and negative. The absolute value of the smallest root decreases as α becomes more negative [36]. In fact, there is a root for any $\alpha<0$, i.e., any $\kappa>4$: if $-1<\alpha<0$, then the asymptotic value of $M(\alpha;2;|z|)$ is negative and so it must possess a root.

In summary, we find that theories having potentials V_κ with $0<\kappa<1$ are stable, with minima lying at spacelike values of B^μ. Theories with $1\leqslant\kappa<2$ and $2<\kappa\leqslant 4$ are unstable, while the case $\kappa=2$ is trivial. Stability is restored for $\kappa>4$, and the vacuum value of B^μ becomes timelike. A timelike vacuum value for B^μ may in fact be favored because the potentials leading to this form of symmetry breaking are more relevant.

The potentials for $0<\kappa<1$ are discussed in Ref. [11]. The hypergeometric functions $M(\kappa-2;2;z)$ have minima with $z<10$ for nearly all κ, and the values of the Kummer functions at these minima are typically also less than 10. However, as $\kappa\to 1$ the potential evolves into an unstable inverted parabola, and so both the location of the minimum and its value diverge in this limit. In the timelike range $\kappa>4$, the potential may possess multiple local minima at negative values of z. However, the exponential damping factor $e^{-|z|}$ in Eq. (9) ensures that the one with the smallest $|z|$ is always the global minimum. As κ increases, the wavelength of the oscillations in $V_\kappa(z)$ decreases, and the location of the minimum is pushed to smaller values of $|z|$. This location may be calculated numerically, and it is roughly given by $z_{\min}\approx-6/(\kappa-3)$. The value $V_{\kappa,\min}$ of V_κ at $z_{\min}$ is consistently close to $V_{\kappa,\min}\approx-0.1g\Lambda^2(\kappa-3)$.

4. Features and implications

We have shown that B^μ must develop a Lorentz-violating vacuum expectation value in any nontrivial stable theory. There are many potential implications of this scenario. An immediate one concerns the interpretation of the excitations about the vacuum. Denoting the vacuum value as $\langle B^\mu\rangle=b^\mu$, we may parametrize B^μ as

$$B^\mu=(1+\rho)b^\mu+A^\mu, \tag{11}$$

where $b^\mu A_\mu=0$. Defining $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, the kinetic term for A^μ is found to be $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. Moreover, at lowest order only fluctuations in ρ cause changes in the potential term in the energy, so there is no mass term for A^μ. We therefore can identify A^μ with the photon field.

The notation in Eq. (11) is chosen to match that of Ref. [19], which provides a general description of the

NG modes associated with spontaneous Lorentz violation and presents the complete effective action for A^μ in various space–times. In this context, excitations around the vacuum of the field A^μ are the NG modes associated with spontaneous Lorentz breaking, while vacuum excitations of ρ are the NG modes for spontaneous diffeomorphism breaking. The masslessness of the photon follows directly from this interpretation as a consequence of the breaking of Lorentz invariance[3] rather than the existence of gauge symmetry. The effective action also contains higher-order corrections to conventional electrodynamics that could be sought in experimental tests. The superficially nonrenormalizable couplings are suppressed by powers of Λ and vanish in the continuum limit.

Since at leading order the potential A^μ satisfies the orthogonality condition $b^\mu A_\mu = 0$, the equivalent conventional electrodynamics must be defined in an axial or generalized axial gauge. One check on the quantum equivalence between electrodynamics and the theory (1) at leading order is provided by a comparison of the corresponding transverse propagators. In fact, the Euclidean propagator for electrodynamics subject to the gauge condition $b^\mu A_\mu = 0$ is[4]

$$D_F^{\mu\nu}(x-y) = -\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \times \frac{1}{p^2}\left[\delta^{\mu\nu} + \frac{b^2}{(b\cdot p)^2} p^\mu p^\nu - \frac{b^\mu p^\nu + b^\nu p^\mu}{(b\cdot p)}\right]. \tag{12}$$

Within the subspace of propagating modes, this is equivalent to the corresponding propagator for the theory (1).

The formulation of the theory (1) involves three propagating modes, and three modes also appear after the spontaneous Lorentz breaking. Two are photon modes contained in A^μ. The third mode is massive, with excitations that change the value of B^2 and hence the potential. The curvature at the global minimum of the potential determines the mass of the fluctuations of B^2 about its vacuum value $b^\mu b_\mu = b^2$. This curvature is always proportional to $g\Lambda^2$. However, g must be small for the linearized calculations to be valid, while the natural physical cutoff scale Λ is expected to be very large, possibly of the order of M_P. It is therefore reasonable to expect that the particles associated with the fluctuations of B^2 are unobservable in a low-energy theory. A large value of κ implies both small b^2 and a large mass for these particles, so weak Lorentz violation is naturally associated with unobservability of the massive mode.

For $\kappa > 4$, the potential remains finite at infinite positive timelike values of B^2. The energy density required to shift the expectation value of the field arbitrarily far from its vacuum value therefore remains finite. However, if Λ is of the order of M_P, this energy density is proportional to gM_P^4. This is suppressed by only one power of g relative to the naive scale of the cosmological constant. The energy density required to generate these large field values exists only in the early Universe, when high-temperature corrections are expected to restore the broken Lorentz symmetry.

Another interesting feature of the timelike case is the inverse variation with κ of the locations of the minima of the V_κ potentials, $z_{\min} \approx -6/(\kappa-3)$. Suppose the underlying theory contains a sum of nonpolynomial interactions V_κ with values of κ ranging to a maximum $\kappa_{\max}$ and with coefficients for the different V_κ potentials controlled by the details of the fundamental physics. When the spontaneous Lorentz breaking is studied in a lower-energy effective field theory with a smaller value of the cutoff, then the effective potential is dominated by those V_κ with κ in the vicinity of $\kappa_{\max}$ because these potentials grow the most rapidly as the cutoff decreases. The magnitude of the Lorentz-violating vector b^μ is then proportional to $1/\sqrt{\kappa_{\max}}$. Since $\kappa_{\max}$ represents the maximum in a potentially large collection of κ values, it can naturally be big. This could provide a partial explanation for the small size of any Lorentz violation in nature.

Additional physical implications of our scenario can be explored by extending the theory (1) to include couplings between B^μ and one or more other fields. For example, introducing a Dirac fermion field ψ offers various possibilities for interactions between B^μ and fermion bilinears. Note that gauge-noninvariant couplings are acceptable here, unlike the usual case of quantum electrodynamics (QED), because the ini-

[3] In the context of electrodynamics without physical Lorentz violation, this interpretation has a long history. See, for example, Ref. [37].

[4] For a discussion of the propagator in axial gauges see, for example, Ref. [38].

tial theory (1) has no gauge invariance. Whatever the nature of the other fields being introduced, Lorentz-violating terms for them appear following spontaneous Lorentz violation when B^μ is replaced with its vacuum value b^μ in the interactions. If the additional fields are identified with ones in the SM or in gravity, the resulting Lorentz violation is contained in the Standard-Model Extension (SME) [8,9]. For example, a simple choice of interaction is $\mathcal{L}_a \propto B_\mu \bar{\psi}\gamma^\mu\psi$, paralleling the usual QED current coupling. When B_μ acquires a vacuum value, this interaction generates the usual coupling of A_μ to the current along with a coefficient for Lorentz violation of the a_μ type in the minimal Lorentz-violating QED extension. For a single fermion, a constant coefficient a_μ is unobservable, but when fermion flavor changes are present coefficients of this type can produce observable effects [39,40]. More exotic couplings could also be countenanced, such as an axial-vector coupling $\mathcal{L}_b \propto B_\mu \bar{\psi}\gamma_5\gamma^\mu\psi$ [13]. When Lorentz symmetry is spontaneously broken, this induces a coefficient for Lorentz violation of the b_μ type in the SME, along with a remnant interaction involving the fluctuations about b^μ. Note that all these couplings are known to be renormalizable at one loop [41]. If the massless excitations are identified with the photon A^μ as above, then a novel coupling to the photon arises.

Many other types of couplings for B^μ can also be considered. Although beyond our present scope, it would be of definite interest to explore features introduced by derivative couplings within this framework. These could provide insight about the structure of higher-derivative terms in the effective action and hence about the causality and stability of theories with Lorentz violation [13], and they could have a bearing on predicted Lorentz-violating effects within the photon sector [42]. It would also be of interest to investigate gravitational couplings, including background spacetimes [9]. Incorporating gravity typically makes RG calculations of the type adopted here impractical, but comparatively simple cases such as conformally flat backgrounds may be tractable. In any event, the occurrence of spontaneous Lorentz violation as a necessary feature in the above stable and relevant theories with nonpolynomial potentials suggests that Lorentz violation might indeed be generic in a large class of underlying theories at the Planck scale.

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